

# Face Recognition in Frequency Domain

Gaiga Matthew Ruangmei<sup>1</sup>, Amit Kaul<sup>2</sup> and R. Nath Sharma<sup>3</sup>

<sup>1,2,3</sup>Electrical Engineering Department, National Institute of Technology, Hamirpur H.P-177005  
E-mail: <sup>1</sup>matti44448888@gmail.com, <sup>2</sup>amitkaul9@gmail.com, <sup>3</sup>r.nath1964@gmail.com

**Abstract**—This paper proposes six face recognition algorithms namely Principal Component Analysis (PCA), Two Dimensional Principal Component Analysis (2DPCA), Two Dimensional Two Directional Principal Component Analysis (2D)<sup>2</sup>PCA, Fourier Magnitude (FM-PCA), Fourier Magnitude (FM-2DPCA) and Fourier Magnitude (FM-(2D)<sup>2</sup>PCA). Face recognition in Spatial Domain using PCA, 2DPCA and (2D)<sup>2</sup>PCA can tolerate different facial expression of invariant illumination with improved performance over the former algorithms. For real time application where mostly facial angle rotation are greater than 60° i.e. intra-pose variations, Spatial Domain Face recognition algorithm performance reduces because of the invisible feature. Frequency domain technique is the solution to the described problem along with additional advantages like ability to distinguish using magnitude and phase spectrum and easiness to go back and forth from spatial domain to frequency domain.

**Keywords:** PCA, 2DPCA, (2D)<sup>2</sup>PCA, FM-PCA, FM-2DPCA, FM-(2D)<sup>2</sup>PCA, Normalized Eigenvectors

## 1. INTRODUCTION

Initially Principal component analysis was introduced for Statistical analysis but later emerged as the global algorithm for object recognition because of its simplicity [1]. In face recognition PCA loses image details resulting from concatenation process [7]. To improve the performance of PCA a new technique called Two Dimensional PCA i.e. (2DPCA) is introduced which employs directly on two dimensional data without concatenation at the cost of more coefficient requirement for image representation. However 2DPCA loses the covariance information between different local geometric structures in the image while PCA preserves the information which is important for recognition [7]. In order to suppress the large coefficient requirement and attain the same or increased accuracy level of 2DPCA, (2D)<sup>2</sup>PCA is introduced by incorporating a Two Directional PCA into 2DPCA [10].

PCA, 2DPCA and (2D)<sup>2</sup>PCA in spatial domain suffers from the adverse effect of intra class pose variation which degrades the performance of algorithm [9]. In order to obtain a more efficient image representation frequency domain is introduced which computes the Fourier magnitude of PCA, 2DPCA and (2D)<sup>2</sup>PCA [9]. In frequency domain Normalized Eigenvectors are computed from the Fourier magnitude covariance matrix rather than the simple Eigenvectors as in spatial domain and

corresponding to it a transformation matrix is obtained to avoid loss of image generality [11]. Fourier Magnitude feature subspaces hold another key advantage. They are shift invariant, as a direct result of the properties of Fourier transform [8]. If the image is shifted in the spatial domain, that shift will translate into a linear phase change in frequency domain and not in its magnitude. This makes Fourier Magnitude subspaces robust to errors in registration, where the input images are not correctly centered which could cause significant recognition errors [8].

Euclidean distance or the Frobenius distance is used for classification for both the algorithm in spatial and frequency domain. This paper is focused in improving the accuracy level of frequency domain algorithm. The remaining parts of the paper are organized as follows. In section 2 face recognition in spatial domain is described. In section 3 face recognition in frequency domain is described. Experimental results on face data bases of NITH, ORL and Yale are presented in section 4. We have Conclusion in section 5. And finally Acknowledgement in section 6.

## 2. FACE RECOGNITION IN SPATIAL DOMAIN

### 2.1 Principal Component Analysis (PCA)

Principal component analysis (PCA) is a well-known feature extraction and data representation technique widely used in the areas of pattern recognition, computer vision and signal processing, etc. It extracts relevant information from confused data sets [1]. Roughly speaking PCA tries to find the axes with maximum variances where the data is most spread (within a class, since PCA treats the whole data set as one class).

Computation of Eigen faces:

Loading Training Images

Step 1: Obtain the face images (which should be centered)  
 $I_1, I_2, \dots, I_M$  (training images)

Step 2: Represent every image  $I_i$  as a vector  $\Gamma_i$

Step 3: Compute the average face vector  $\Psi$

$$\Psi = \frac{1}{M} \sum_{i=1}^M \Gamma_i \tag{1}$$

Step 4: Subtract the mean face from every image matrix individual which produce the Scatter matrix

$$\Phi_i = \Gamma_i - \Psi \tag{2}$$

Step 5: Compute the covariance matrix C

$$C = \frac{1}{M} \sum_{i=1}^M \Phi_i \Phi_i^T = AA^T \tag{3}$$

where  $A = [\Phi_1 \Phi_2 \dots \Phi_M]$  MXN image dimension;

M= No. of Class (i.e. Person) x No. of sample per class

Step 6: The goal is to compute the eigenvectors  $u_i$  of  $AA^T$ . The matrix  $AA^T$  is very large and a full eigenvector calculation is impractical. So we do Eigen decomposition of  $A^T A$  instead of  $AA^T$ .

The eigenvectors  $u_i$  and Eigenvalues  $\lambda_i$  of C are such that  $Cu_i = \lambda_i u_i$  (3.1)

These are related to the eigenvectors  $v_i$  and Eigen values  $\lambda'_i$  of the matrix  $D=A^T A$  in the following way:

$$Dv_i = \lambda'_i v_i, A^T A v_i = \lambda'_i v_i, AA^T A v_i = \lambda'_i A v_i \tag{4}$$

$$CAv_i = \lambda'_i A v_i, C(A v_i) = \lambda'_i (A v_i), C u_i = \lambda'_i u_i \tag{5}$$

So the Eigen vectors and Eigen values of C can be computed as

$$u_i = Av_i, \lambda_i = \lambda'_i \tag{6}$$

Thus,  $AA^T$  and  $A^T A$  have the same eigenvalues and their eigenvectors are related as follows:

$$u_i = Av_i, \lambda_i = \lambda'_i$$

Step 7: Keep only  $K$  eigenvectors (corresponding to the  $K$  largest eigenvalues). Eigenvectors may not be equal to the zero vector. A nonzero scalar multiple of an eigenvector is equivalent to the original eigenvector. Hence, without loss of generality eigenvectors are often normalized to unit length. So Normalized Eigenvector is optional if there is no loss of image generality.

Projection on Eigen faces

$$\Omega_j = \sum_{j=1}^K w_j u_j; w_j = u_j^T \Phi_j \tag{7}$$

we call the  $u_j$ 's Eigen faces

Face Recognition Using Eigen faces

Given an unknown test face image  $\Gamma_{test}$  (centered and of the same size like the training faces) follow these steps:

$$\text{Normalize } \Phi_{test} = \Gamma_{test} - \Psi \tag{8}$$

Project on the Eigen space

$$\Omega_{test} = \sum_{i=1}^K w_i u_i; w_i = u_i^T \Phi_{test} \tag{9}$$

$$\text{Find } e_r = \min (\|\Phi_i - \Phi_{test}\|) \tag{10}$$

The distance  $e_r$  is called Euclidean distance within the face space (differences).

### A. Two Dimensional Principal Component (2DPCA)

In the PCA-based face representation and recognition methods, the 2D face image matrices must be previously transformed into 1D image vectors column by column or row by row. However, concatenating 2D matrices into 1D vectors often leads to a high-dimensional vector space, where it is difficult to evaluate the covariance matrix accurately due to its large size and the relatively small number of training samples [7]. Furthermore, computing the eigenvectors of a large size covariance matrix is very time-consuming.

To overcome those problems, a new technique called 2-dimensional principal component analysis (2DPCA) was recently proposed, which directly computes eigenvectors of the so-called *image covariance matrix* without matrix-to-vector conversion. Because the size of the image covariance matrix is equal to the width of images, which is quite small compared with the size of a covariance matrix in PCA, 2DPCA evaluates the image covariance matrix more accurately and computes the corresponding eigenvectors more efficiently than PCA. It was reported in that the recognition accuracy on several face databases was higher using 2DPCA than PCA, and the extraction of image features is computationally more efficient using 2DPCA than PCA. However, the main disadvantage of 2DPCA is that it needs many more coefficients for image representation than PCA [7]. Here we consider that the 2DPCA is working in row direction of image.

Consider  $A$  an  $m$  by  $n$  random image matrix.

Let  $X \in R^n$  be a matrix with orthonormal columns,  $n \geq d$ . Image matrix

$$A_k = [A_k^1 A_k^2 \dots A_k^m]$$

and average of image matrix be

$$\bar{A} = [\bar{A}^1 \bar{A}^2 \dots \bar{A}^m]$$

Projecting  $A$  onto  $X$  yields an  $m$  by  $d$  matrix  $Y=AX$ . In 2DPCA, the total scatter of the projected samples is used to determine a good projection matrix  $X$ .

The *image covariance matrix*  $\mathbf{G}$  which is an  $n$  by  $n$  matrix is nonnegative definite matrix. Suppose that there are  $M$  training face images, denoted by  $m$  by  $n$  matrices

$$A_k (k = 1, 2, \dots, M) \text{ and } \bar{A} \text{ denote the average image as } \bar{A} = \frac{1}{M} \sum_{k=1}^M A_k$$

Then  $\mathbf{G}$  can be evaluated by

$$\mathbf{G} = \frac{1}{M} \sum_{k=1}^M (A_k - \bar{A})^T (A_k - \bar{A}) \quad (11)$$

It has been proven that the optimal value for the projection matrix  $X_{opt}$  is composed by the orthonormal eigenvectors  $X_1 X_2 \dots X_d$  of  $\mathbf{G}$  corresponding to the  $d$  largest eigenvalues, i.e.  $X_{opt} = [X_1 \dots X_d]$  Because the size of  $\mathbf{G}$  is only  $n$  by  $n$ , computing its eigenvectors is very efficient.

### B. Alternate 2DPCA

Let  $A_k = [(A_k^1)^T (A_k^2)^T \dots (A_k^m)^T]^T$  and

$\bar{A} = [(\bar{A}_k^1)^T (\bar{A}_k^2)^T \dots (\bar{A}_k^m)^T]^T$  be the image column matrix. 2DPCA and alternative 2DPCA only works in the row and column direction of images respectively. That is, 2DPCA learns an optimal matrix  $\mathbf{X}$  from a set of training images reflecting information between rows of images, and then projects an  $m$  by  $n$  image  $\mathbf{A}$  onto  $\mathbf{X}$ , yielding an  $m$  by  $d$  matrix  $\mathbf{Y} = \mathbf{A}\mathbf{X}$ . Similarly, the alternative 2DPCA learns optimal matrix  $\mathbf{Z}$  reflecting information between columns of images, and then projects  $\mathbf{A}$  onto  $\mathbf{Z}$ , yielding a  $q$  by  $n$  matrix  $\mathbf{B} = \mathbf{Z}^T \mathbf{X}$ .

The image covariance matrix  $\mathbf{G}$  can be obtained from the outer product of row vectors of images, assuming the training images have zero mean, i.e.  $\bar{A} = (\mathbf{0})_{m \times n}$ . For that reason, we claim that original 2DPCA is working in the row direction of images. Then alternative covariance matrix  $\mathbf{G}$  can be evaluated by

$$\mathbf{G} = \frac{1}{M} \sum_{k=1}^M (A_k - \bar{A})(A_k - \bar{A})^T \quad (12)$$

Similarly, the optimal projection matrix can be obtained by computing the eigenvectors  $Z_1 Z_2 \dots Z_q$  Corresponding to the  $q$  largest eigenvalues, i.e.  $Z_{opt} = [Z_1 \dots Z_q]$ . Because the eigenvectors only reflect the information between columns of images, we say that the alternative 2DPCA is working in the column direction of images.

### Two Directional Two Dimensional PCA, (2D)<sup>2</sup>PCA

A simultaneously way of presenting 2DPCA and Alternate 2DPCA is (2D)<sup>2</sup>PCA which uses the projection matrices  $\mathbf{X}$

and  $\mathbf{Z}$  of 2DPCA and Alternate 2DPCA respectively. (2D)<sup>2</sup>PCA Preserves the accuracy of 2DPCA but eliminates the large number of coefficient requirement of 2DPCA [10]. Suppose we have obtained the projection matrices  $\mathbf{X}$  and  $\mathbf{Z}$ , projecting the  $m$  by  $n$  image  $\mathbf{A}$  onto  $\mathbf{X}$  and  $\mathbf{Z}$  simultaneously, yielding a  $q$  by  $d$  matrix  $C_{trn}$

$$C_{trn} = \mathbf{Z}^T \mathbf{A} \mathbf{X}$$

The matrix  $C_{trn}$  is also called the coefficient matrix in image representation, which can be used to reconstruct the original image  $\mathbf{A}$ . When used for face recognition, the matrix  $C_{trn}$  is also called the feature matrix. After projecting each training image  $A_{trn}$  ( $trn = 1, 2, \dots, M$ ) onto  $\mathbf{X}$  and  $\mathbf{Z}$ , we obtain the training feature matrices  $C_{trn}$  ( $trn = 1, 2, \dots, M$ ). Repeating the same for test image  $A_{test}$  we get the test feature matrix  $C_{test}$ . Then the nearest neighbor classifier is used for classification. Here the distance between  $C_{trn}$  and  $C_{test}$  is defined by

$$d(C_{trn}, C_{test}) = \|C_{trn} - C_{test}\| = \sqrt{\sum_{i=1}^q \sum_{j=1}^d (C_{trn}^{(i,j)} - C_{test}^{(i,j)})^2} \quad (13)$$

## 3. FACE RECOGNITION IN FREQUENCY DOMAIN

In this section, we develop Fourier magnitude versions of PCA, two-dimensional PCA algorithms (2DPCA), (2D)<sup>2</sup>PCA by using the Fourier magnitudes of the image pixels for feature extraction instead of the raw pixel values. Instead of the scatter matrix in equation 2 of spatial domain we will use the image matrix in Frequency domain [11].

### 1. Fourier Magnitude PCA

Although PCA is a widely used technique for face recognition, it has major drawbacks of losing the image details, having a large time complexity and suffering from the adverse effect of intra class pose variations [9]. To overcome the intra class drawback in PCA, Fourier magnitudes are employed for the feature extraction step of the PCA algorithm in FM-PCA.

The algorithm computation in frequency domain is same as spatial domain except the extra calculation of normalized eigenvectors and its use in place of the simple eigenvectors to avoid any intrusion of information from the original image.

### 2. Fourier Magnitude 2DPCA

The huge number of coefficient requirement of 2DPCA in spatial domain will exist in frequency domain FM-2DPCA also but the accuracy level gets boosted up because of the two

new modified steps as compared to the spatial domain [7] & [9].

The Fourier magnitudes for the pixels of an image are first computed. The magnitudes of the Fourier coefficients for the  $i^{th}$  training image  $I^i$  can be represented as an  $M \times N$  matrix given by

$$I_{FM}^{(i)} = [I_{FM}^{(i)}(u, v)]_{M \times N}$$

Similarly the FM version of a test image

$$I_{FM}^{(test)} = [I_{FM}^{(test)}(u, v)]_{M \times N}$$

This Fourier coefficients of training and test images are to be used in place of the difference matrix which were used in spatial domain algorithms. The Fourier magnitude matrices of the training images are also used to obtain the covariance matrix of FM-2DPCA as

$$C_{FM-r2DPCA} = \frac{1}{K} \sum_{i=1}^K (I_{FM}^{(i)} - \overline{I_{FM}})^T (I_{FM}^{(i)} - \overline{I_{FM}}) \quad (14)$$

where  $K$  is the number of training samples and  $\overline{I_{FM}}$  is the average of the training images and is given by

$$\overline{I_{FM}} = \frac{1}{K} \sum_{i=1}^K I_{FM}^{(i)} \quad (15)$$

After calculating the eigenvalues and eigenvectors from the Covariance matrix the eigenvectors are normalized and rearranged in descending order of the corresponding eigenvalues. A transformation matrix

$V_{FM} = [V_{FM1} V_{FM2} \dots V_{FM\beta}]$  is obtained from the Normalized Eigenvectors of  $C_{FM-r2DPCA}$ . The Projection matrix for the  $i^{th}$  training sample and test sample are given respectively as,

$$Z_{FM}^{(i)} = I_{FM}^{(i)} V_{FM} \quad (16)$$

$$Z_{FM}^{(test)} = I_{FM}^{(test)} V_{FM} \quad (17)$$

For classification module of the FM-2DPCA algorithm, matrix similarity measures, such as the Euclidean or Frobenius distance is used,

$$d_{FM}(Z_{FM}^{(i)}, Z_{FM}^{(test)}) = \sqrt{\sum_{i=1}^q \sum_{j=1}^d (Z_{i^{th}}^{(i,j)} - Z_{test}^{(i,j)})^2} \quad (18)$$

The wanted person is said to be identified if the subject (i.e. distance) of the training sample whose feature matrix has the shortest distance from the test image feature matrix.

#### I. Fourier Magnitude Two Directional 2DPCA

It is the highest accuracy algorithm with reduced coefficient requirement and preserving the accuracy attained and also removing the intra pose problem [7], [9] & [10]. In FM – (2D)<sup>2</sup>PCA algorithm the row-directional transformation covariance matrix is computed using  $C_{FM-r2DPCA}$  and its column-directional counterpart is computed as

$$C_{FM-c2DPCA} = \frac{1}{K} \sum_{i=1}^K (I_{FM}^{(i)} - \overline{I_{FM}})(I_{FM}^{(i)} - \overline{I_{FM}})^T \quad (19)$$

The eigenvalues and eigenvectors of  $C_{FM-r2DPCA}$  and  $C_{FM-c2DPCA}$  are then computed. Then the Eigenvectors are normalized and arranged in descending order of the corresponding eigenvalues. Then, the row-directional and column directional transformation matrices

$$V_{FM} = [V_{FM1} V_{FM2} \dots V_{FM\beta}] \quad (19.1)$$

and

$$U_{FM} = [U_{FM1} U_{FM2} \dots U_{FM\alpha}] \quad (19.2)$$

are obtained using sufficient numbers of the Normalized Eigenvectors of  $C_{FM-r2DPCA}$  and  $C_{FM-c2DPCA}$ , respectively. Then the Projection matrix for the  $i^{th}$  training sample and test sample are given respectively as,

$$Z_{FM}^{(i)} = U_{FM}^T I_{FM}^{(i)} V_{FM} \quad (20)$$

And

$$Z_{FM}^{(test)} = U_{FM}^T I_{FM}^{(test)} V_{FM} \quad (21)$$

## 4. EXPERIMENTAL RESULTS

Simulations are performed on a 1.70 GHz Intel(R) Premium(R) 3558U CPU with 4GB RAM and Windows 8 operating system. Simulations are carried out using MATLAB. The algorithm are tested on benchmark face database ORL (Olivetti Research Lab, Cambridge Univ), YALE and NITH (National Institute of Technology, Hamirpur) face database. In ORL database there are 40 individual classes with 10 samples each. The first 5 samples of each classes are used for training images and the remaining for test images. In YALE and NITH database there are 50 individual classes with 12 samples each. The same treatment is done for YALE and NITH as with the ORL database. All the facial images from the three database has illumination variation. The YALE database are mainly concerned with different facial expression but with no intra pose variation of all the classes whereas the NITH facial database has maximum intra pose variation as compared to the remaining two database. The two tables given below are the results achieved from the MATLAB simulation experiments for all the Database with the algorithms.

Accuracy achieved:

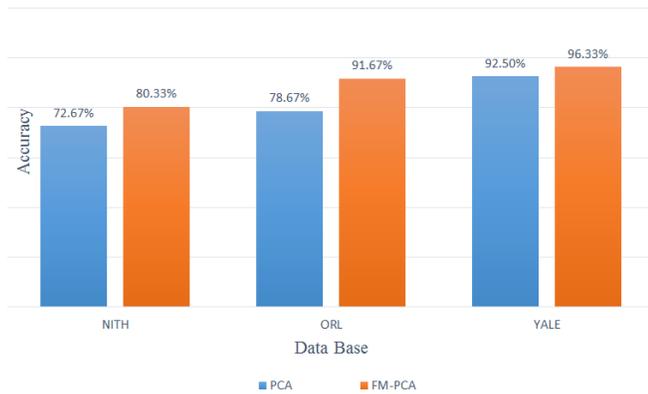
| Spatial Domain Methods | NITH    | ORL | YALE    |
|------------------------|---------|-----|---------|
| PCA                    | 72.667% | 78% | 92.50%  |
| 2DPCA                  | 78.667% | 90% | 95.667% |
| (2D) <sup>2</sup> PCA  | 78.667% | 90% | 95.667% |

Accuracy achieved:

| Frequency Domain Methods | NITH    | ORL    | YALE    |
|--------------------------|---------|--------|---------|
| FM-PCA                   | 80.333% | 81%    | 96.333% |
| FM-2DPCA                 | 91.667% | 96.50% | 100%    |
| FM-(2D) <sup>2</sup> PCA | 91.667% | 96.50% | 100%    |

Given below is the accuracy graph of all the Database which a comparison of Spatial Domain and Frequency Domain for PCA and 2DPCA algorithm. Since (2D)<sup>2</sup>PCA has the same accuracy level of 2DPCA so it is not plotted though (2D)<sup>2</sup>PCA requires lesser number of computational time as compared to the 2DPCA algorithm because (2D)<sup>2</sup>PCA requires lesser number of coefficient as compared 2DPCA.

Algorithm Accuracy PCA v/s FM-PCA



Algorithm Accuracy 2DPCA v/s FM-2DPCA



### 5. CONCLUSION

The enhanced accuracy level in frequency domain is achieved due to substitution of two new steps:

- a) Scatter image matrix from equation (2) replaced with the Image matrix.
- b) The ability to distinguish separately using the magnitude spectrum in the frequency domain algorithm.

As an overall result we have seen that the YALE Database which has only different facial expression under invariant illumination environment has the highest accuracy in both the domain analysis, but under intra pose variation with mild and extreme level of face angle rotation in ORL and NITH database respectively, the accuracy level in Spatial domain is low. When we analysed the same in Frequency domain the efficiency level in improved.

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